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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, MARCH 2022
THIRD YEAR [BATCH 2019-22]

Date : 28/02/2022 **MATHEMATICS**
Time : 11am-1pm **Paper** : MTMA CC12 Full Marks : 50

Group A

Answer all questions. Maximum one can score is 30.

1. Find the mass and center of mass of a plate on the triangle $0 \leq x \leq 1$, $0 \leq y \leq x$ whose density is $\rho(x, y) = y^2 - x$. [3]

2. For $f(x, y) = \ln(x^2 + xy + 1)$, find a unit vector direction in which the value of f is not changing at the point (1, 1). [3]

3. Find the circulation of the vector field $\vec{F}(x, y, z) = \langle 2xy, xz, y\rangle$ along the line segment from (0, 1, 0) to (4, 3, 4). [4]

4. Find the equation of tangent plane to the surface $r(s, t) = \langle s^2, 2st, t^3\rangle$ at (4, 4, 1). [4]

5. If H be a homogeneous function of degree n in x and y and if $u = (x^2 + y^2)^{-\frac{n}{2}}$ and if H possesses continuous first order partial derivatives, show that
$$\frac{\partial}{\partial x}(H\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(H\frac{\partial u}{\partial y}) = 0.$$
[5]

6. For the function $f(x, y) = 3x^2 + 2y^3 - 6xy$, find the critical points and classify them as minima, maxima, or saddle points. [5]

7. Evaluate the integral $\int\int_R (y + x)dA$, where R is the region bounded by the lines $y = x$, $y = 2 + x$, $y = -x$ and $y = 3 - x$. (Hint: You may make the calculation simpler by taking suitable change of variables.) [6]

8. Use Stokes' theorem to evaluate $\int \vec{F}.d\vec{r}$ where $\vec{F} = 2x^2\hat{i} - 4z\hat{j} + xy\hat{k}$ and C is the circle of radius 1 at $x = -3$ and perpendicular to the x-axis. C has a counter clockwise rotation if you are looking down the x-axis from the positive x-axis to the negative x-axis. [6]

Group B

Answer any 2 questions. **[5 x 2 = 10 marks]**

9. Obtain Fourier Series corresponding to the function $f(x) = x$ on $[-\pi, \pi]$. [5]

10. Obtain Fourier Cosine Series corresponding to the function $f(x) = \sin x$ on $[0, \pi]$. [5]

11. Obtain Fourier Sine Series corresponding to the function $f(x) = \cos x$ on $[0, \pi]$. [5]

Group C

Answer any 2 questions. **[5 x 2 = 10 marks]**

12. Obtain a nonsingular transformation that will reduce the quadratic form $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ into normal form. [5]

13. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{bmatrix}$. [5]

14. Find orthogonal matrix P such that $P^{-1}AP$ is diagonal matrix, for $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$. [5]

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